

Observables in Relativistic Quantum Mechanics

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We propose a quantum clock synchronization protocol in which Bob makes a remote measurement on Alice's quantum clock via a third qubit acting as its proxy. It is shown that the resulting correlations are dependent on the choice of the hypersurface along which Bob's measurement of the proxy is deemed to collapse the entangled state vector. A proper characterization of observables in relativistic quantum mechanics is therefore constrained by relativistic covariance as well as causality.

Whereas the unitary evolution of states in quantum mechanics (QM) is given by covariant equations of motion [1], the collapse of the state vector is manifestly not. Traditionally, the observable is a Hermitian operator defined on an equal-time hypersurface in the inertial reference frame of the observer. This hypersurface corresponds to the simultaneous state-vector collapse of the observed state. Although this does not necessarily violate causality, it is problematic from the viewpoint of special relativity since simultaneity is not Lorentz invariant [2]. Covariant formalisms to describe state vector reduction have been discussed in Refs. [3]. The problem of how operations in QM are constrained by causality has been considered by a number of authors [4]. Issues pertaining to the causality and localizability of superoperators on bipartite systems have been dealt with by Beckman et al. [2] and Eggeling et al. [5].

The process of narrowing down of the probability distribution of the measured observable that accompanies a measurement is called reduction [6]. Whether reduction reflects only a change in our knowledge of the system, or an objective alteration of the system such as in the manner described in Refs. [7–9], or an abrupt nonlocal collapse of the wavefunction precipitated by a classical observer, are important and difficult questions of physical and interpretational interest that lie beyond the scope of the present work. We are concerned only with the effective picture of how the quantum description of a system should change in response to measurements in a special relativistic setting.

To this end, we consider two (for simplicity:) stationary observers, Alice (located at $x = a$) and Bob (at $x = b$), sharing pairs of entangled qubits. Alice's qubit ("A") is a quantum clock, i.e., it has nondegenerate eigenstates, governed by the Hamiltonian $H = \omega(|0\rangle\langle 0| - |1\rangle\langle 1|)$ with energy eigenvalues $\pm\omega$ (setting $\hbar = 1$). The set-up is similar to the quantum clock synchronization protocol [10], except (for one) that Bob's qubit ("B") states remain degenerate.

At time $t = 0$, Alice and Bob share the singlet state

$$|\Psi(0)\rangle_{AB} = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle). \quad (1)$$

After lapse of time t (which is also proper time in Alice's rest-frame), the above state has evolved to:

$$\begin{aligned} |\Psi(t)\rangle_{AB} &= \frac{1}{\sqrt{2}}(e^{i\omega t}|01\rangle - e^{-i\omega t}|10\rangle) \\ &= |+\rangle|\psi^+(t)\rangle - |-\rangle|\psi^-(t)\rangle, \end{aligned} \quad (2)$$

where $|\pm\rangle \equiv (1/\sqrt{2})(|0\rangle \pm |1\rangle)$ and $|\psi^\pm(t)\rangle \equiv \pm i \sin \omega t |\pm\rangle \mp \cos \omega t |\mp\rangle$. Suppose Alice measures in the $+/-$ basis at time t_a . Thereby she prepares Bob's qubit in one of the states $|\psi^\pm\rangle$ instantaneously as seen from her frame. Subsequently, Bob measures qubit B in the $+/-$ basis. Alice classically communicates her result to Bob. Working on a large number of pairs of qubits, Bob expects to verify the joint CB (and hence AB) measurement probabilities

$$\begin{aligned} P(+, +) &= P(-, -) = \frac{1}{2} \sin^2 \omega t_a, \\ P(+, -) &= P(-, +) = \frac{1}{2} \cos^2 \omega t_a. \end{aligned} \quad (3)$$

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We note that the correlations in Eq. (3) depend only on time t_a of Alice's measurement and are independent of the choice of the collapse hypersurface. Therefore, as such QCS is unsuitable to test for possible dependence of correlations on the reduction hypersurface.

Alternatively, Bob can make a remote measurement on Alice's qubit via a proxy. By prior arrangement, Alice uses a third particle, a degenerate qubit denoted C . This qubit is made to locally interact with A , the outcome of which is represented by the unitary operation U given by:

$$(\alpha|+\rangle + \beta|-\rangle)_C \otimes |\pm\rangle_A \xrightarrow{U} (\alpha|\pm\rangle + \beta|\mp\rangle)_C \otimes |\pm\rangle_A, \quad (4)$$

where $|\alpha|^2 + |\beta|^2 = 1$. Per the protocol, Alice begins with C in the state $|+\rangle$. After the interaction, she classically sends Bob the proxy qubit C . The state of the 3-qubit system now is:

$$|\Upsilon\rangle_{CAB} = |+\rangle|+\rangle|\psi^+(t)\rangle - |-\rangle|-\rangle|\psi^-(t)\rangle, \quad (5)$$

in place of Eq. (2). Now Bob first measures C at time t_1 in the $+/-$ basis. Thereby he disentangles all three qubits and instantaneously knows whether Alice's qubit is left in the state $|+\rangle$ or $|-\rangle$ without Alice's classical communication.

In traditional parlance, Bob's measurement on C collapses (or "decoheres"/"reduces") the CAB system along his equal-time hypersurface. In Figure 1, this is the surface 1, which intercepts A 's worldline at $t_a = t_1$, i.e., at event (a, t_1) . For each shared pair of qubits, if he finds C (and hence, A) in the state $|\pm\rangle$, he concludes that B is left in the state $|\psi^\pm\rangle$. After performing a second measurement, on B , he expects to find the correlations in Eq. (3) with $t_a = t_1$. On the other hand, a third observer, whose equal-time hypersurface passing through event (b, t_1) is given by surface 2 in Figure 1, would expect that Alice's qubit disentangles at event (a, t_2) and that Bob would find correlations given by Eq. (3) with $t_a = t_2$, rather than $t_a = t_1$. Similarly, correlations with $t_a = t_3$ in Eq. (3) are predicted to be found by an observer whose equal-time hypersurface is given by surface 3 in Figure 1, and so on. This implies that in order to ensure the Lorentz invariance of Eq. (3), a unique hypersurface should correspond to Bob's measurement, which can be inferred by Bob from the observed correlations. Tests of this kind can be implemented at present, e.g., using rapidly moving detectors (cf. Refs. [11]).

We wish to stress that the preceding result does not violate causality, since no classical signal is transferred along the hypersurface. Only correlations are affected. Indeed, in one sense the dependence of correlations on the choice of hypersurface, and thence the need for a covariant description of observables based on a singled out hypersurface, brings quantum measurement closer to the spirit of Special Relativity. A more detailed discussion of the above experiment and repercussions of our result for the epistemology and models of quantum measurement and for quantum information and relativistic quantum mechanics will be dealt with in future works.

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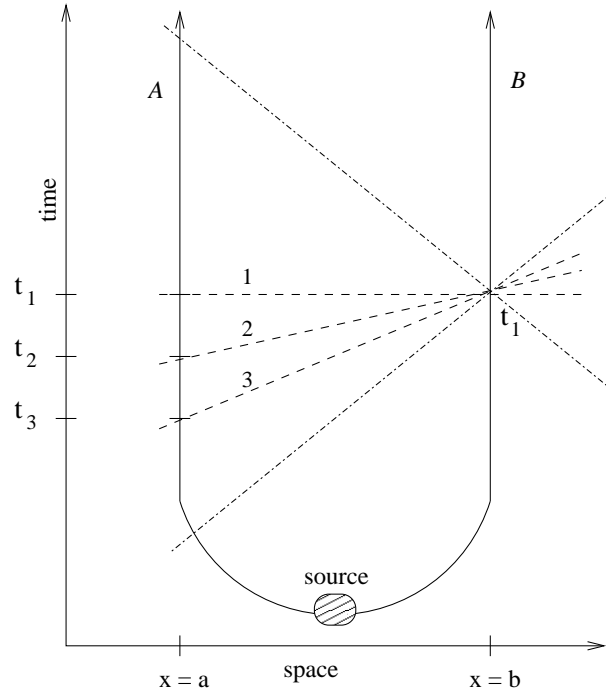


FIG. 1. A and B are two entangled particles, located at $x = a, b$, respectively, whose most probable worldlines are indicated by the bold lines. The dashed lines 1, 2 and 3 are possible spacelike collapse hypersurfaces corresponding to Bob's measurement on Alice's proxy qubit at event (b, t_1) , and intercepting Alice's worldline at times t_1 , t_2 and t_3 . The choice of the collapse hypersurface determines the phase of Alice's quantum clock at the instant of disentanglement, and hence the correlations between qubits B and C observed by Bob. The dash-dotted lines represent the future and past lightcones of event (b, t_1) .